Elective Course 2 (EC -2)

MATH PEDAGOGY

GREY NOTES

B.Ed. F.Y. SEM – 2

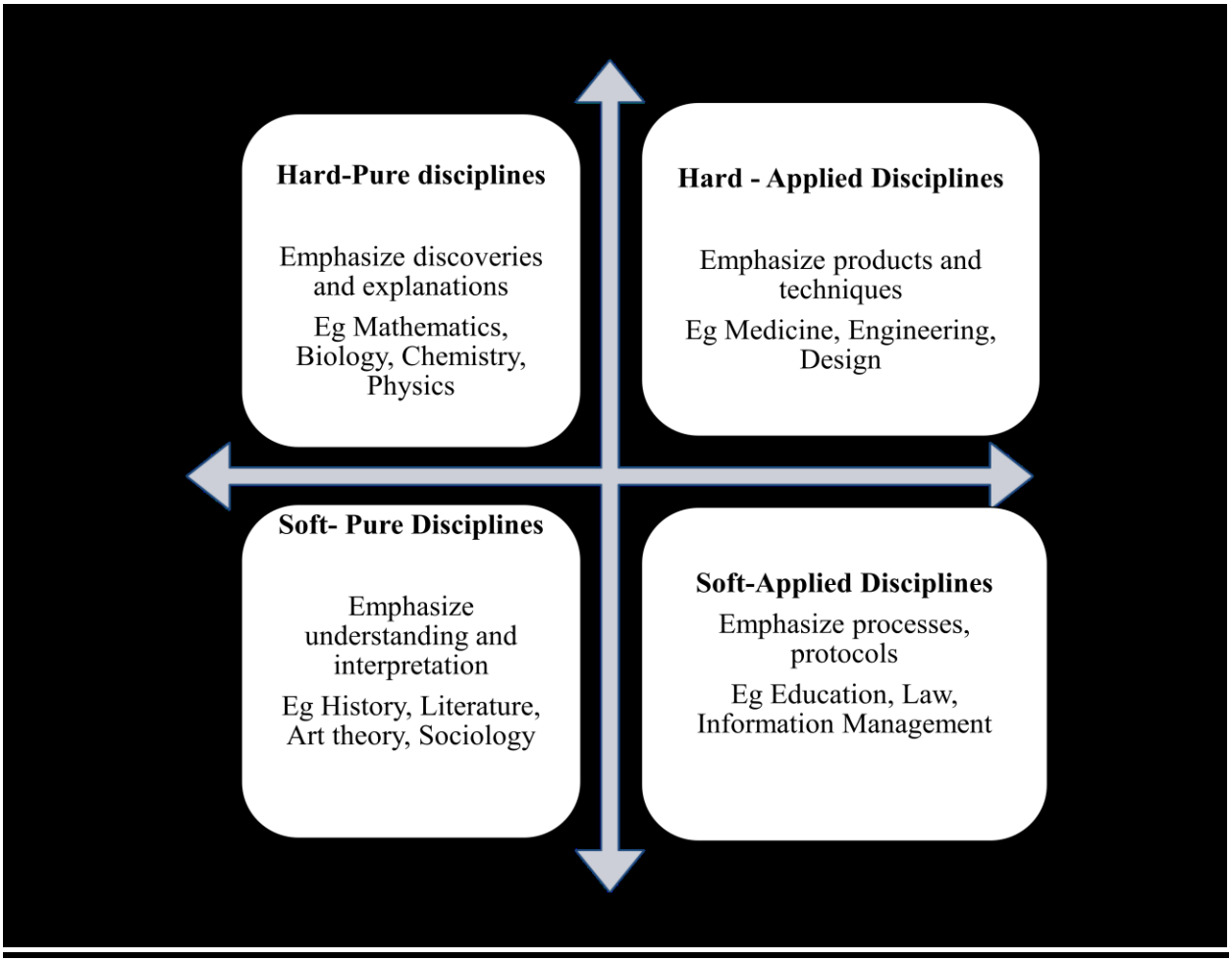
Dnyan Ganga Education Trust's College of Education (B.Ed.)

Thane

INDEX

Unit 1: Basics of Academic Disciplines

1A- Meaning of Academic Disciplines



Hard-Pure disciplines involve general areas of human understanding and are clustered around limited number of problems. The nature of knowledge in these disciplines is cumulative and concerned with universal phenomena. The result of such knowledge is discovery of something new or expansion of already existing knowledge. Just like a crystal grows as more and more molecules add on to it, so is it in case of this group of disciplines. As new knowledge keeps adding, the older form of knowledge is enhanced. For example, consider the knowledge about an atom. As research on the atom progressed, our knowledge about atoms made incremental progress. The relationship between the knowledge seeker and knowledge is unbiased and very objective. There are very definite criteria to verify knowledge in such disciplines. There is a high degree of consensus over significant questions. For example if two scientists are studying the effect of temperature on the states of matter, their results will be similar no matter which parts of

the world they perform their experiments in. Academic communities in hard-pure disciplines are well organized, their work is quite competitive and publication rates are high.

Hard-Applied Disciplines are involved in purposive work. The emphasis is on application of theories resulting in creating techniques and products. These disciplines are practical in nature and are concerned with solving problems, addressing challenges and mastering the environment around us. The focus is on application and hence heuristic approaches find more importance in such disciplines. They use both quantitative and qualitative approaches. The criteria for judging the product of such disciplines are functional. Such disciplines result in new techniques and products being created. Engineering, for example, is a hard-applied discipline which draws from Mathematics, Physics and Chemistry. Clinical Medicine is a hard-applied discipline dependent upon Biology and Chemistry. The ethos in such disciplines is entrepreneurial and dominated by professional values. Patents are submitted for publication.

Soft-Pure Disciplines stress on understanding and interpretation of phenomena. Knowledge in these disciplines is reiterative which means there may be repetition of knowledge when examined in different situations. These disciplines are concerned with particular happenings rather than general occurrences. Unlike hard sciences, here data is qualitative. The researcher and knowledge share a personal relationship. There can be different views regarding what verification of data. Subjectivity can be high when interpretations are made. There is no definiteness as to what significant questions are to be answered. Anthropology and History are some examples of disciplines in this type. The academic communities of such disciplines tend to be less structured compared to those from hard-pure disciplines. Publication rate is also lower.

It is interesting to note that while a discipline like Sociology is a soft-pure discipline, Sociometrics, a subfield of Sociology, is hard-pure.

Soft-Applied Disciplines emphasize processes and protocols. These are functional and utilitarian in nature. They are concerned with the enhancement of professional practice. Often their status is uncertain. They also appear to be dominated by intellectual fashions. They use a mix of qualitative and quantitative data for their growth. Case studies form an important tool in such disciplines. Law and Education are examples of soft-applied disciplines. Education depends upon other soft disciplines like Psychology, Sociology, Philosophy. Publication rates in these disciplines are low.

A brief summary of the above discussion is given below

Discipline Types	**Nature of knowledge**	**Objects of Inquiry in discipline type**	**Enquiry procedures**	**Results of Research**	**Culture of the discipline type**	**Disciplinary areas**
Hard-Pure	Cumulative and concerned with phenomena	Concerned with universal things and quantities	Clear criteria for knowledge verification	Discovery and theories	Competitive, high number of publications, task oriented	Natural Sciences
Hard-Applied	Purposive and pragmatic	Concerned with mastery Of physical environment	Qualitative and quantitative approaches, use of Heuristic methods	Patents, products	Entrepreneurial, Dominated by professional values, role oriented culture	Science based Professions
Soft-Pure	Reiterative, holistic knowledge	Concerned with particular events	Qualitative methods, there is dispute over criteria For knowledge verification	Results in understanding and interpretation	Individualistic, loosely cohesive communities, low publication rates	Humanities and Social Sciences
Soft - Applied	Functional and utilitarian knowledge	Concerned with enhancement of professional practice	Use of case studies	Results in protocol and procedures	Outward looking, dominated by intellectual fashions, power oriented,	Social Professions

Thus the understanding of Biglan-Becher typology gives an overview of how different disciplines are similar and how they differ. One also sees how a particular group of disciplines has somewhat similar characteristics with respect to research carried out or publications made. The Biglan - Becher groupings can be a tool to promote intra- as well as interdisciplinary dialogue through recognition of the differences and similarities present in the various groups. Research in Education must take this typology into consideration as Education (from the soft applied type) often relies on disciplines like Psychology and Sociology (soft-pure). A good understanding of the typology of disciplines will result in better quality of research. Interdisciplinary studies too need to look at this typology carefully.

Interdisciplinary and multidisciplinary teaching and learning: meaning, significance and role of the institution

Interdisciplinary and multidisciplinary approaches make educational experiences authentic. Curricula that reflect real life are more meaningful to students. Students are able to see natural and logical connections that cut across content areas. The traditional approach to learning organizes content into compartments based on subject matter boundaries. In interdisciplinary and multidisciplinary approaches, content revolves around questions, themes, problems and projects.

Meaning of Interdisciplinary teaching and learning

Heidi Jacobs defines interdisciplinary learning as "a knowledge view and curriculum approach that consciously applies methodology and language from more than one discipline to examine a central theme, issue, problem, topic, or experience"

Keith Barton and Lynn Smith suggest that interdisciplinary learning is especially important in the early grades so as to "provide authentic experiences in more than one content area, offer a range of learning experiences for students, and give students choices in the projects they pursue and the ways they demonstrate their learning."

Barton and Smith explain that interdisciplinary units enable teachers to use classroom time more efficiently and address content in depth, while giving students the opportunity to see the relationship between content areas and engage in authentic tasks.

1 B - Classification of academic disciplines: becher -biglan typology (pure-hard, pure-soft, applied- hard, applied-soft types) with emphasis on nature of knowledge in each type.

There is no definite system with respect to classification of disciplines. Some experts classify disciplines based on the focus of their content as Arts and Humanities, Socials Sciences, Natural Sciences, Mathematics and Business Studies. In the early 1970s Anthony Biglan carried out a study to investigate the faculty's judgment about similarities and differences between several academic fields. These perceptions were classified as

i. Pure vs applied
ii. Hard vs soft (or paradigmatic vs non paradigmatic disciplines)
iii. Concerned with life systems vs those not concerned with life systems.

Let us examine the meaning of these terms

Classification on basis of research undertaken

Pure Disciplines: Pure disciplines refer to those disciplines that tend towards fundamental research. There is systematic observation of phenomena solely for the purpose of discovering unknown facts which may develop into theories. The product of these disciplines is some kind of new knowledge. Simply put a pure discipline is a discipline that involves study purely for the sake of knowledge and not for its application. Some examples are Pure Mathematics, Pure Physics, Pure Chemistry, Pure History. To elaborate, Pure Mathematics solves problems, finds facts and answers questions that don't depend on the world around us, but on the rules of Mathematics itself.

Applied Disciplines: Applied disciplines relate existing knowledge to real world situations. These disciplines make significant contributions to the world by articulating the theoretical foundations in their field of study. For example Human Resource Development is an applied discipline that draws heavily from pure disciplines like Psychology and Sociology. Engineering is an applied science dependent on the pure sciences of Mathematics and Physics.

Classification on basis of data involved

Hard disciplines: Disciplines that tend to use quantitative data, tend to be predictive and use experimental methods are classified as a hard disciplines. Eg: Physics, Chemistry, Engineering, Computing are all examples of hard disciplines as they deal with quantitative data. They use

experimental methods to build their repository of knowledge. Braxton (1995) represents the hard disciplines as being characterized by greater concern for career development and cognitive goals (such as the learning of facts and concepts)

Soft Disciplines: Soft disciplines are those disciplines that rely on qualitative data. They generally do not use experimental methods and hence cannot make conclusive predictions concerning the future. Examples of soft disciplines are Languages, Law, Anthropology and Education. The soft disciplines as being characterized by greater concern for general education development, character development, critical thinking and 'scholarly' activities (such as the reading of research articles).

Disciplines are also classified as those dealing with life systems or living beings as against those that deal with inanimate objects. Examples of the former are Biology, Psychology and those of the latter type are Physics, Mathematics, Geology.

A study by Smart and Ethington (1995) reports the opinions on the goals of undergraduate education from over 4000 university faculty members who regularly taught undergraduate students. The conclusions were that soft and applied disciplines place greater emphasis on knowledge acquisition, and hard disciplines have more concern for knowledge application. Knowledge integration and application were both perceived to be more important in the applied disciplines than in the pure.

While Biglan's work concentrated on the cognitive dimension of disciplines, Becher in 1989 called attention to the social dimensions of academic disciplines. From this emerged the Biglan-Becher typology of academic disciplines. According to this typology, four main types of groups are possible

1. Hard and Pure disciplines
2. Hard and Applied disciplines
3. Soft and Pure disciplines
4. Soft and Applied disciplines

1 C- Place of mathematics in the present school curriculum

Curriculum includes all those activities, experiences and environment which the child receives during his educational career under the guidance of educational authorities. Thus curriculum is the total education of the child.

Curriculum touches all the aspects of the life of the pupils- the need and interest of pupils environment which should be educationally congenial to them, ways and manners in which their interest can be kindled warmed up, the procedures and approaches which cause effective learning among them, the social efficiency of the individual and how they fit in with the community around.

Lindsay says, "Mathematics is the language of physical science and certainly no more marvelous language was ever created by the mind of the man."

More above ***Galileo*** has expressed has views as "Mathematics is language in which god has written the universe"

Being so important "What place should be given to mathematics in the curriculum?" In school Kothari Commission has explained about placing mathematics as a compulsory subject up to higher secondary or tenth standard and has said, "Mathematics should be made a compulsory subject for the students of 1st to X standard, as a part of general education."

But some people lay more emphasis on making it an optional subject after eight-standard, therefore various reasons were framed against this proposal:

- It is very difficult subject and its learning requires a sharp brain and intelligence, as many children will face difficulties for gaining the knowledge.
- It is only an imagination the mental abilities, discipline, culture, social and moral developments can be done by mathematics.
- The numbers of failures in mathematics in high school examination are more as compared to that of other subjects.
- Every student can't become an engineer or a technician, then what is the necessity of mathematics for all.

Reasons for Keeping Mathematics in School Curriculum

- Mathematics is the Basis of all Sciences: The different branches of science likewise-Physics, Chemistry, Astronomy, Biology, Medical Science, Geology, and Astrology etc. are the important subjects which are based on mathematics for e.g. area, volume, weight, density, number of atoms and electrons, medicines all are related to mathematical study.
- Mathematics is related to Human Life: Right from getting up in the morning till going to bed we need the help of mathematics. For purchasing, planning our day, each and every aspect involves the use of mathematics. Today in the modern age, the knowledge of mathematics is essential and more important in one form or the other.
- Mathematics Generates Logical Attitude: Mathematics give training to different faculties of mind. In order to solve a mathematical problem a child has to think logically. Every step is, related to other step on the basis of some logic with which child develops his mental abilities and it further effects his intellectual development.
- Mathematics Provides a Definite way of Thinking: The children who study mathematics develop attitude with which they learn to work systematically, regularly and properly. Along with this it also develops a logical thinking in them.
- Mathematics is an exact science: By the study of mathematics child develops the attitude to accept the knowledge of mathematics in an exact form. All mathematical concepts, formulae, facts are related to exactness and thus it removes the feeling of doubt. For example; $4 + 4 = 8$. Which cannot be 7 or 9 etc.
- Mathematics provides opportunity to develop mental abilities of the child.

- Mathematics deals with significant, abstracts and consistent structures.
- Mathematics is the study of sets and structures.
- Mathematics helps in character formation as well as morality.
- It develops the characteristic of discipline.
- The language of mathematics is universal.
- Knowledge of mathematics is useful in the study of other school subjects.

All great educationalists like Herbert, Pestolozzi etc. has accepted mathematics as a symbol of human development. Accepting mathematics as a best means of intellectual and cultural developments, these educationists placed mathematics on the top in the curriculum. Thus we can give certain logical points regarding mathematics as a compulsory subject. These are as follows:

- If mathematics is not given an important place in the curriculum then students would not get any opportunity for mental training and in the absence of which their intellectual development might be affected.
- For gaining the knowledge of mathematics no innate power is required, which is separate from ability of study of other subject.
- Training of reasoning, thinking, discipline, self-confidence and emotions are developed in students by mathematics.

A good student of mathematics must be in position to appreciate the precision, logic, sharpness and beauty of its peculiar language. Several students lose interest in the subject as they are symbolism. Thus, it is the duty of the teacher of mathematics to create in him an interest towards its language.

UNIT- 2 INTRODUCTION TO THE TEACHING OF MATHEMATICS

2 A- Meaning nature and scope of mathematics

NATURE OF MATHEMATICS

a. Meaning:

- In Greek it means ―inclined to learn
- Oxford Dictionary, ―the branch of science concerned with number, quantity and space.
- Locke, ― a way to settle in the mind a habit of reasoning
- Ancient Hindus referred to mathematics as, ―Ganita—the science of calculation.
- Courant and Robbins, ―Mathematics is an expression of the human mind that reflects the active will, contemplative reason and the desire for aesthetic perfection. Its basic elements are logic and intuition, analysis and construction, generality and individuality.

The following conclusions can be made related to the meaning of mathematics:

- It is a science of number and space
- Has its own language in terms of signs, symbols, terms, operations etc.
- Uses/Requires intuition, logic, reasoning, analysis, construction, generality and individuality.
- Helps in drawing conclusions and interpreting various ideas and themes.
- It is suited for dealing with abstract concept of any kind.
- Helps to solve problems of daily life.
- Has an aesthetic value and helps to admire the beauty of nature.

b. Branches of mathematics:

Pure Mathematics and Applied Mathematics

c. Nature of Mathematics:

Mathematics relies both on logic and creativity.

It is pursued for a variety of:

- Practical Purposes i.e. how mathematics applies to their work?
- Intrinsic Interests i.e. Essence of mathematics lies in its beauty and intellectual challenge.

The nature of mathematics can be also discussed in terms of:

i. ***Science of logical reasoning****:* In mathematics the results are developed through a process of reasoning.

Reasoning in mathematics possesses a number of characteristics such as,

- Simplicity
- Accuracy
- Certainty of Results
- Originality
- Verification
- Conclusions follow naturally from the facts when logical reasoning is applied to the facts.

ii. ***Mathematical Language and Symbolism****:*

- It has its own unique language and symbols.
- Mathematical language and symbols cut down on lengthy statements.
- It helps in the expression of ideas and concepts in exact form.
- It is free from verbosity, helps to point out clear and exact expression of facts.
- E.g ―Writing (a-b) 2 = $a^2 – 2ab + b^2$ in words.
- Symbols which are peculiar and unique to mathematics

iii. Values Inculcated through mathematics*:*

- Utilitarian or Practical Value: Refer from math method text as discussed.
- Intellectual Value: Imagination, memory, observation, concentration, creativity, logical, thinking and reasoning are developed through mathematics.
- Disciplinary Value: Punctuality, Neatness, cleanliness, habit of paying attention, regular study habits through homework and drill are developed in students via mathematics.

Pure Mathematics

- Arithmetic
- Algebra
- Geometry

Applied Mathematics

Relates to a wide range of studies with a wide use in empirical sciences
Aesthetic Value: It has regularity, symmetry, order, a specific arrangement, which lends beauty to art forms. (Architecture, art, drawing, dance, drama etc)

d. Importance of Mathematics in Curriculum:

This can be studied through the aims and objectives of teaching mathematics.

Aims of Teaching Mathematics:

- They enable the students to acquire mathematical knowledge, skills, develop interests and attitudes.
- The various aims are as follows:
- Utilitarian aim
- Disciplinary aim
- Cultural aim
- Social Aim
- Aesthetic Aim
- Vocational aim
- Inter-disciplinary aim.
- To provide a clarity about fundamental concepts and processes of mathematics.
- To create in pupils an enduring interest for the subject.
- To develop in pupils a taste for and confidence in mathematics.
- To develop in pupils accuracy and efficiency.
- To acquaint pupils with relation of mathematics with their present as also their future life.
- To develop habits such as regularity, practice, patience, self-reliance and hard work.
- To acquaint pupils with mathematical language and symbols.
- To prepare pupils for learning of mathematics of higher classes.
- To initiate and develop required discipline in the learner's mind.

Objectives of Teaching Mathematics:

The objectives of teaching mathematics at the entire school stage are classified as:

1. **Knowledge and Understanding:**

The student acquires Remembering and understanding of –

- Language of mathematics
- Concepts of number, measurement and direction.
- Development and history of mathematics.
- Interrelationship between different branches and topics of mathematics.
- Nature of the subject.

2. **Skill: Helps the students to develop the given skills**

- To use mathematical language.
- Acquire speed, neatness, accuracy and Precision.
- Technique of Problem-solving.
- To estimate check and verify results.
- Perform calculations orally and mentally.
- Drawing of geometrical figures.
- Use of mathematical tables, tools and apparatus.

3. **Application:**

Helps the students to apply knowledge, understanding and skills as follows:

- Able to solve problems on their own.
- Use of concepts and processes in daily life.
- Ability to analyze, draw inferences and generalize.
- To use mathematical knowledge in other subjects.
- Apply mathematics in his or her vocation.

4. **Attitude:**

The students of mathematics develop or must develop the following attitudes:

- Learn to analyze problems.
- Habit of systematic thinking and objective reasoning.
- Discover solutions and proofs due to independent efforts.
- Verifies the results.
- Develops mathematical perspectives and outlook towards society.
- Appreciates logical, critical and independent thinking.

5. **Appreciation and Interest:**

These are also long term objectives and are as follows:

- Role of mathematics in daily life.
- Aesthetic nature of mathematics
- Recreational value of mathematics.
- Interest in learning the subject
- Appreciates the power of computation.

2 B- Aims and objectives of teaching Mathematics at secondary and higher secondary levels (NCF 2009)

INTRODUCTION:

- National Curriculum Framework (NCF) 2009 is an attempt to improve teacher education by Government of India and to prepare ideal, innovative, humane and affectionate teachers.
- The framework is an endeavour of the National Council for Teacher Education to encourage views on the qualitative and quantitative improvements that could be achieved in educating teachers at school, graduate, post-graduate, doctoral and post-doctoral levels.
- It outlines an inspiring vision of teacher education, covering both pre-service and in-service teacher education, as well as preparation of teacher educators.
- Mathematics comes from the Greek word "máthema", which means ―inclined to learn.
- It is a science of number and space.
- Helps to solve problems of daily life.

AIMS for secondary level:

1. To provide a clarity about fundamental concepts and processes of mathematics.

2. To create in pupils an enduring interest for the subject and its application.

3. To prepare pupils for learning of mathematics of higher classes

4. To develop in pupils a taste for and confidence in mathematics.

5. To acquaint pupils with mathematical language and symbols.

6. To consolidate a high level of mathematical literacy.

7. To develop skills required for analysis, problem solving and interpretation of data.

OBJECTIVES for secondary level:

- Students begin to perceive the structure of mathematics as a discipline. They become familiar with the characteristics of mathematical communication: carefully defined terms and concepts, the use of symbols to represent them, precisely stated propositions, and proofs justifying propositions. These aspects are developed particularly in the area of geometry.
- Students develop their facility with algebra, which is important not only in the application of mathematics, but also within mathematics in providing justifications and proofs.
- At this stage, students integrate the many concepts and skills that they have learnt into a problem-solving ability.
- Mathematical modelling, data analysis and interpretation taught at this stage can consolidate a high level of mathematical literacy.

- Individual and group exploration of connections and patterns, visualisation and generalisation, and making and proving conjectures are important at this stage, and can be encouraged through the use of appropriate tools that include concrete models as in Mathematics laboratories and computers.

AIMS for higher secondary level:

1. To develop in pupils accuracy and efficiency.

2. To acquaint pupils with relation of mathematics with their present as also their future life.

3. To develop habits such as regularity, practice, patience, self-reliance and hard work.

4. To acquaint pupils with mathematical language and symbols.

5. To initiate and develop required discipline in the learners mind.

6. To create awareness about a range of its application in various fields thereby increasing career opportunities.

OBJECTIVES for higher secondary level:

- To provide students with an appreciation of the wide variety of the application of Mathematics, and equip them with the basic tools that enable such application.
- The rapid explosion of Mathematics as a discipline, and of its range of application, favours an increase in the breadth of coverage.
- Such increase must be dictated by mathematical considerations of the importance of topics to be included.
- Topics that are more naturally the province of other disciplines may be left out of the Mathematics curriculum.
- The treatment of topics must have an objective, that is, the communication of mathematical insights and concepts, which naturally arouse the interest and curiosity of students.

LINK referred:https://pcer.ac.in/wp-content/uploads/2016/02/Maths-meth.pdf

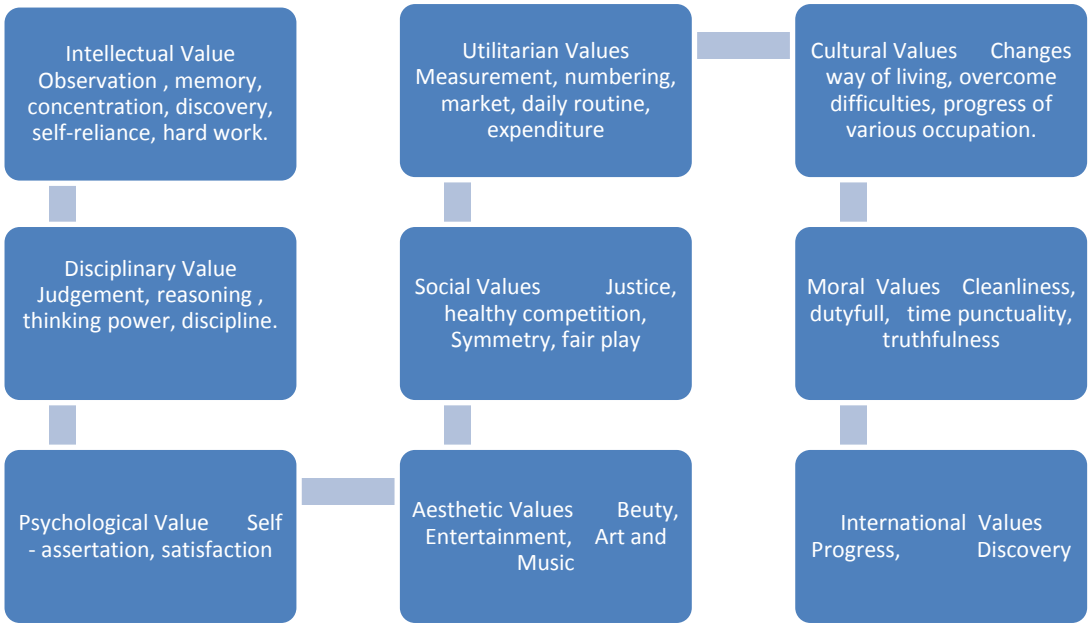
2 C- Values of teaching Mathematics – (Ref: Dr. A.K. Kulshrestha)

Introduction:

Values give meaning and strength to a person's character by occupying a central place in his life.

Values are guiding principles of life which are conducive to all round development.

Therefore values reflect one's personal attitudes, judgments, decisions, choices, behavior, relationships, dreams and vision. Therefore mathematics plays an important role in progress of society.



1. **Intellectual value :**

There is no other subject in curriculum like mathematics which make student's brain active. Problem solving help us in developmental of mental faculties, each problem of mathematics possesses such a sequence which is necessary for constructive and creative process. In this way, all mental abilities of a child are developed through mathematics.

2. **Utilitarian or Practical Value :**

We use mathematics knowledge in our daily routine, house, outside, market, income—expenditure etc.

In the absence of mathematical knowledge a person can neither treat his family well nor can he face his social duties.

3. Disciplinary Value :

Mathematics develop child personality with qualities like concentration, truthfulness, seriousness etc. mathematics is the only subject whose knowledge develops the habit of hard work, concentration, well organized and clarity in the students.

4. Moral Value :

Through mathematics study child develops qualities of cleanliness, reality, punctuality, truthfulness, honesty, self-control, self-reliance, patience, listens to others and respect them etc.

5. Social Value :

In order to live a social life, its knowledge is needed because the give and take process, business and industry depend upon the knowledge of mathematics.

6. Cultural Value :

The history of mathematics presents the image of culture of different nations. The person becomes cultured, critical observer, logical thinker and proper knowledge of mathematics changes the mind of the person.

7. Aesthetic Value :

Mathematician discovers something new with the help of mathematical laws, facts, theorems and principles, a sense of joy is developed in his mind, he realizes the aesthetic aspect of his findings or research.

8. Vocational Value :

The aim of education is to learn living and make learner self-dependent. All vocations like engineering, technology, information technology, management are prestigious and important. The knowledge and training of this vocation is possible only through mathematics. Almost every vocation needs knowledge of mathematics.

9. Psychological Value :

Mathematics education is also useful from the point of view of psychological aspects. Mathematics fulfills the psychological needs of the children. In mathematics emphasis is given on operations and drill work so that its knowledge becomes more solid as well as durable.

10. Scientific Attitude :

The knowledge of mathematics trains the children in attempting the problems according to a definite and distinct procedure which may be called as the scientific method. Training which a child receives in studying mathematics can be applied to solve the problem arising in new situations.

UNIT : 3 Essentials of Teaching Mathematics and Curriculum Transaction

3 A - Maxims of teaching:

Maxims of Teaching are the universally facts found out by the teacher on the basis of experience. They are of universal significance and are trustworthy. The knowledge of different maxims helps the teacher to proceed systematically. It also helps to find out his way of teaching, especially at the early stages of teaching.

The different maxims of teaching are briefly explained below.

Known to Unknown:

This maxim is based on the assumption that the student knows something. We are to increase his knowledge and widen his outlook. We have to interpret all new knowledge' in terms of the old. It is said that old knowledge serves as a hook on which the new one can be hung. Known is trustworthy and unknown cannot be trusted. So while teaching we should proceed from known and go towards unknown. For example, while teaching any lesson, the teacher can link the previous experiences of the child with the new lesson that is to be taught Teaching of Math.

Simple to Complex:

Class-room teaching is formal where the teacher tries to teach and the students try to learn things. In this process of teaching-learning, the teacher should see that simple things are presented first to the students. That way they will start taking interest. Once they become interested, thou gradually complex type of things can also be learnt by them. By learning simple things, they feel encouraged and they also gain confidence. On this basis, they become further receptive to the complex matter. On the other hand, if complex types of things are presented to the learner first, he become, upset, feels bored and finds himself in a challenging situation lot which he is not yet ready being immature and unripe.

Gradually more difficult items of learning may be presented to the students. It will smoothen teaching being done by the teacher and make learning convenient and interesting for the students.

For example, while teaching Math, addition and subtraction should be taught first and complex multiplication and division may be taken afterwards.

Concrete to abstract:

Concrete things are solid things and they can be touched with five senses. But abstract things can only be imagined. So it is rather difficult to teach the children about abstract things. The students are likely to forget them soon. On the other hand, if we teach the students with the help of concrete objects, they will never forget the subject matter.

For example when we teach counting to the students we should first examine concrete nouns like, laptop, book, Pen etc. and then proceed to digits and numbers. The stars, the moon, the sun etc. being taught first whereas the abstract thing:, like planet, satellites etc. should be taught afterwards.

Analysis to synthesis.

Analysis means breaking a problem into its convenient parts while synthesis means grouping of these separated parts into one complete whole. A complex problem can be made simple and easy by dividing into different parts.

"Analysis is the approach for understanding and synthesis is for fixation." Analysis of a sentence' is taught to students, that helps the students to understand the different parts of a sentence. Later on, synthesis of sentences should be taught.

Particular to General:

While teaching, the teacher should first of all take particular statements and then on the basis of those particular cases, generalization should be made. Suppose the teacher is teaching Math, he should first of all give a few examples of math problem and then on the basis of those make them generalize to math problems.

Empirical to Rational.

Empirical knowledge is based on observation and firsthand experience. II is particular concrete and simple. We can see, feel and experience it on the other hand; rational is based on our arguments, and explanation. The stage of arguments is the last whereas seeing things or feeling them is the first stage. Empirical is less general statements whereas rational is more general statements. So the safe approach in teaching is that we should proceed from empirical to rational. It is a journey from less mental maturity to more mental maturity.

Induction to Deduction

Induction means drawing a conclusion from a set of examples whereas deduction is its opposite. The teacher should proceed from induction to deduction.

Psychological to Logical.

While teaching, the teacher should first keep in mind the interest, aptitudes, capacities, development level etc. of the children during selection of subject matter and then on to its logical arrangement.

In teaching English, the structures are selected as per needs and requirements of the students and then arranged in a logical way. The psychological appeal of the thing is more important at the early stages. Then the' logic behind it should be seen.

Whole to Part.

In teaching, the teacher should try to acquaint the child with the whole lesson first and then the different portions of it may be analyzed and studied intensively. This principle holds good while teaching a thing to the small children. At the early stages, the child loves to speak full sentences because in daily life situations, full sentences are used. The child should be given a full sentence. Then he may have full familiarity with the different words contained in that sentence. Later he may have the knowledge of words. Then he will have the knowledge of different letters forming the words.

Suppose a poem is to he taught to the students. They should be acquainted with the full poem first. Gradually they may be asked to grasp the poem stanza by stanza In the case of average students, their first attempt may be on full stanza, taking it as a whole and then to the different lines con I. lined in the stanza as parts. It will help the teacher to teach better and the learners to learn things conveniently.

Definite to Indefinite.

In teaching, definite things should be taught first because the learner can easily have faith in them. Then afterwards he should give the knowledge of indefinite things. Definite things, definite rules of grammar help the learner to have good knowledge. Gradually he can be taught about indefinite things.

The above given maxims are only hints and guidelines for the teacher, especially at the initial stages. He may use them if he finds some of them useful in his teaching situations. In some situations of class-room teaching, he may not use them if he feels so. The teacher should keep the maxims in his hand and he should remain their master. Then only the different maxims will remain tools and yield better result.

3B-Approaches of curriculum construction: curriculum organization – topical, concentric.

Concentric Approach

In this approach the topics will find a place in different classes of different years of a course in a progressive manner. The content will be included from simple to complex as the pupils understand the content according to capabilities that present in chronological and mental ages.

The concentric approach is a way of organizing a curriculum by laying out basic concepts, covering other related material, and then circling back around to the basic concept and filling in more complexity and depth.

Instead of life science, earth science, physics, biology and chemistry being separated and studied in sequence, each year's curriculum revisits the sciences studied earlier.

It's believed that starting with fundamentals that are then regularly revisited, built on, deepened and broadened each time leads to a better understanding of a subject's interconnections.

The organization of curriculum using concentric approach is useful in primary and secondary school levels.

Merits-

1. It proceeds from ‘simple to complex ‘and ‘whole to part’.
2. Greater opportunity for revision of topic.
3. It takes into consideration mental growth of the pupil.
4. Continuity can be maintained. Limitations-

1. Repetition is sometimes cumbersome. Some facts are repeated again and again.

2. The presentation lacks novelty and freshness.

3. Less appealing and fails to arouse interest.

4. Pupil develops a sense of familiarity without the fullness of knowledge.

Topical Approach:

Topical arrangement means that a topic should be finished entirely at one stage. It takes the topic as a unit. Topical arrangement requires that easy and difficult portions of a topic should be dealt with at one stage only which is psychological.

In topical approach all relevant material is covered in linear fashion and concepts are not revisited.

In this system the topic which is dealt with earlier receives no attention later and so there is every likelihood of its being forgotten

They are introduced with a view to make the teaching of the topic complete and thorough. Hence topical method demands that a topic once taken should be finished entirely.

This is not much useful for lower classes.

Merits-

1. Integrated knowledge is imparted to the pupils.
2. In-depth , thorough knowledge of the topic.
3. Pupil’s interest and motivation is sustained.

4. Correlation of subjects.

5. This approach can be adopted according to the age, ability of the students.

Limitations-

The main defect in the topical method is that it introduces in the curriculum a largeness of irrelevant material for which the pupil finds no time and no immediate need or the use of which cannot be appreciated by the pupil at that stage.

3C - Pedagogical Analysis

a) Content Analysis
b) Instructional Objectives
c) Instructional Strategies
• Concept Pedagogical analysis- Meaning and Process
• What is Content analysis and how it is done in Mathematics?
• What are the Instructional Objectives for Mathematics?
• What are the different Instructional Strategies that can be used in Mathematics?

- The word Pedagogy was derived from the Greek words, ped-meaning child and agogus-meaning leader of.
- It refers to passive methods of teaching-learning.
- It mentions that the students are empty vessels and the teacher can pour knowledge into them.
- This approach to learning was called Pedagogy. Thus in pedagogy, the concern is transmitting
- The content and the teacher alone take all decisions about learning.
- Today, however, the term 'pedagogy' has taken a new meaning. Thus the New Meaning of Pedagogy is facilitating the learner in mastering the content.
- Hence Pedagogy means *the art and the science of teaching orLearner focussed education for people of all ages.*

PedagogicalAnalysis

- Why to Teach: Aims/ Learning Outcomes
- What to Teach: Content/Concepts
- How to Teach: Approaches to teaching & learning

In pedagogy, development is based upon a content plan:

- What content needs to be covered?
- How can this content be organized into manageable units or modules?
- How can this content be transmitted in a logical sequence?
- What would be the most effective method for transmitting this content (media)?

Learning Outcomes/Aims Learning Contents/Concepts Connected with:
Actions handling/processes
Educational Resources Teaching & Learning Approach connectedwith contents/concepts/issues

Pedagogical Analysis-Process

Pre-active

- ascertaining entry competence
- stating learning outcomes
- identifying contents + subordinate concepts
- identifying types of learning

Interactive

- Methods/approaches of stimulus presentation
- Eliciting desired learners' response
- Giving suitable feedback

Evaluative

- determining the learning outcome
- setting new objectives of teaching.

Pre-active Stage

- Structure of the subject
- Analysis of curriculum
- Analysis of syllabus
- Comparison of syllabus & textbook
- Units in the text book
- Content analysis

Content analysis

- Identifying contents + subordinate concepts
- Identifying the terms, facts, principles, rules, etc.
- Values and core elements in the unit.

Interactive Stage

Learning Experiences to be provided for:

- Understanding content
- Sensitizing to core elements
- Inculcating Values
- Selecting appropriate methods / approaches of stimulus presentation.
- Eliciting desired learners' response.
- Giving suitable feedback.

Evaluative Stage-Evaluation of learning and teaching,

- If by evaluation is done questioning method

The questions should be based on:

- Content
- Core Elements
- Values
- Generalizations

NPE (1986) with modifications in 1992 incorporates the basic spirit of Article 51A of the constitution **and** emphasizes that**:**

"…**The National System of Education will be based on a national curricular framework which contains a *COMMON CORE* along with other components that are flexible.**

• The Core Elements

1. The history of India's freedom movement,
2. Constitutional obligation,
3. Content essential to nurture national identity,
4. India's common cultural heritage,
5. Egalitarianism, democracy and secularism,
6. Equality of sexes,
7. Protection of environment,
8. Removal of social barriers,
9. Observance of the small family norms and
10. Inculcation of scientific temper.

• Lesson Planning / Structure

- The intentions of the lesson OBJECTIVES AND SPECIFICATIONS
- At the end of lesson what will be the LEARNING OUTCOMES?
- What CONTENT needs to be covered to fulfill these intentions?
- Which TEACHING METHODS are best suited to achieve these intentions?
- Which CLASS MANAGEMENT STRATEGIES will match the students' learning needs?
- What RESOURCES and AUDIO/VISUAL AIDS need to be prepared in advance/be available?
- How will VARIETY OF ACTIVITY be used to sustain student's interest ?
- How will learning outcomes be recorded for EVALUATION ?

Content Analysis

Cognitive Development centered Subjects: Maths, Science, Geography
Affective Development centered Subjects: Languages, History
Psychomotor Development centered Subjects: Physical Education, Music, Drawing, Work Experience/SUPW, Practical

Content Analysis of following components is done

- Terms
- Concepts
- Definitions
- Facts
- Formulae
- Laws
- Conclusions
- Principles
- Generalizations
- Processes
- Procedures

Entry competence

Knowledge of the school subject (Mastery level)

Knowledge of Instructional Objectives and Specifications

Activities and Procedures

Evaluation Technique

Learning outcomes (Aims)
Through pedagogical analysis,

- A student teacher becomes conversant with the objectives of teaching a unit, becomes aware of the relation of these objectives with the core components, can identify essential entry behavior of pupils,

- A student teacher chooses appropriate

Curriculum transaction strategies,
Classroom management techniques
Evaluation strategies are used to evaluate achievement of the pre-determined objectives, sensitizing for the core elements and inculcating values.

As an essential part of the training of teachers, a thoughtful integration of mastery of subject matter, insight gained through pedagogical analysis and the foundation courses, for classroom instruction will improve the QUALITY OF EDUCATION.

The **content analysis** is the structured description of the subject matter that is involved in the activities. This description includes the conceptual structure of the subject matter, its representation systems, its phenomenological analysis and its modelling possibilities. The cognitive analysis is the identification and description of the difficulties that students might face and the errors that they might make while accomplishing the tasks that constitute the activities.

The **instruction analysis** is the description of the activities that are to be proposed to the students, taking into account the types of tasks that emerge from the content analysis, the needs of students (as a consequence of the cognitive analysis), and the materials and resources available.

The **performance analysis** is the description of the students 'cognitive status as a result of the activities. This information feedbacks a new cycle of the didactical analysis. At the end of one cycle (which can be a lesson or a portion of a lesson), the information produced in the performance analysis is used in the formulation of new goals, contents and cognitive status.

UNIT-4 Methods and Techniques of Teaching Mathematics

4A - Learner Centered Methods in Mathematics

- **What is method?** - The word "Method" has been derived from Latin which word means, "Mode" or "Way".
- **What is Method of Teaching?-** "The process of interpreting the world of knowledge to pupils mind is called the method of teaching"

Child Centered or Learner Centered Teaching

- The child occupies a central position.
- Teaching – learning process is geared to the needs, interests, capabilities & requirements of the child. based on psychological principles.
- To develop abilities, skills & discovery attitude amongst the students.
- Teacher is passive.
- Includes, project, laboratory, problem – solving, heuristic, discussion method etc

Methods of Teaching Mathematics

- Lecture Method.
- Demonstration Method.
- Lecture Cum – Demonstration Method.
- Inductive – Deductive Method.
- Analytic – Synthetic Method.
- Laboratory Method.
- Heuristic Method.
- Project Method.

- Problem – Solving Method.

ANALYTIC METHOD

- The world analytic is derived from the world "analysis" which means "breaking up" or "separate things that are together"
- Analysis starts with "what we have to find out & traces the connection between it & data".
- Proceed from "unknown to known".
- From conclusion to hypothesis.
- In this method we start from "what is to be determined" or "what is to be proved".
- It is also called "method of discovery

Used in following conditions:

- To prove any theorem.
- When construction work is to be done in the geometry.
- To find the solution for arithmetic problem.

Application of Analytic Method in Mathematics

Example : If a/b = c/d, then P.T ac $+4b^2$ / bc = c^2 + 4bd/dc.

Solution

To prove this using analytic method, begin from the unknown.

The unknown is ac + $4b^2$/bc = c^2+4bd/dc.

=> ac+ $4b^2$/b = c^2+4bd/d.

=> d(ac+ $4b^2$) = b(c^2+4bd).

=> dac+ $4db^2$ = bc^2+$4b^2$d.

=> dac = bc^2.

=> da = bc.

=> a/b = c/d.

This type of the solution of the problem can be easily obtained by analysis.

Advantages of Analytical Method

- Based on psychological principles.
- Explanatory procedure.
- Creates creativity & originality in the child.
- Develops analytic & reasoning power.
- Develops scientific attitude.
- Analysis is the process of thinking.
- Develops self – confidence & logical abilities in the child.
- Knowledge gained by this method is more solid & durable.
- The child is always curious for attaining new knowledge.
- Formative method & based on inductive reasoning.

Demerits of Analytical Method

- This is a lengthy method.
- It is not possible to acquire speed & efficiency.
- Every teacher cannot use this method successfully.
- Whole syllabus cannot completed in certain period.
- Analytic method is possible only when we have knowledge of known facts & unknown conclusion

SYNTHESIS METHOD

- Reverse of the analytic method.
- Synthesis means " to join separate parts".
- We proceed "from known to unknown" or "start with hypothesis & end with conclusion".
- Method of formulation, recording & presenting concisely the discovered solution omitting the trial & errors.
- Synthesis leads to rote memory & doing by mere.
- Synthesis without analysis is dogmatic, but synthesis is offer analysis has a place in the class – room

Application of Synthesis Method in Mathematics

Example : If a/b = c/d, then P.T ac +$4b^2$ / bc = c2 + 4bd/dc.

Solution :

We known that a/b = c/d.

In this method we start from known & proceed to unknown.

Adding 4b/c on both sides.

a/b + 4b/c = c/d + 4b/c.

=> ac +$4b^2$ / bc = c^2 + 4bd/dc.

Which was to be proved.

Note

No logic or reason is gives here for why 4b/c is added on both sides.

Analysis is the process of discovering the solution & synthesis is the method of setting out the solution in a concise from so as to convince yourself & others.

Advantages of Synthesis Method

- It is a short & quick method.
- It glorifies the memory of the child.
- It formulates records & presents concisely the discovered facts.
- It omits the trials & errors like in analysis.
- The solution in a concise form.
- It is informative method.
- It takes less time.

Demerits of Synthesis Method

- No scope of discovery.
- It leads to rote memory.
- It create many doubts in the mind of the child.
- It does not give full satisfaction to the child.
- No opportunity for developing thinking, reasoning & other mental abilities.

INDUCTIVE METHOD

- Inductive approach is advocated by Pestalaozzi and Francis Bacon
- Inductive approach is based on the process of induction.
- In this we first take a few examples and greater than generalize.
- It is a method of constructing a formula with the help of a sufficient number of concrete examples. Induction means to provide a universal truth by showing, that if it is true for a particular case. It is true for all such cases. Inductive approach is psychological in nature.
- The children follow the subject matter with great interest and understanding. This method is more useful in arithmetic teaching and learning.

Inductive approach proceeds from

- Particular cases to general rules of formulae
- Concrete instance to abstract rules
- Known to unknown

- Simple to complex

Following steps are used while teaching by this method:-

- **Presentation of Examples**
    - In this step teacher presents many examples of same type and solutions of those specific examples are obtained with the help of the student.
- **Observation**
    - After getting the solution, the students observe these and try to reach to some conclusion.
- **Generalization**
    - After observation the examples presented, the teacher and children decide some common formulae, principle or law by logical mutual discussion.
- **Testing and verification**
    - After deciding some common formula, principle or law, children test and verify the law with the help of other examples. In this way children logically attain the knowledge of inductive method by following above given steps.

Example 1:

Square of an odd number is odd and square of an even number is even.

Solution:

Particular concept:

$1^2 = 1$ $3^2 = 9$ $5^2 = 25$ equation 1

$2^2 = 4$ $4^2 = 16$ $6^2 = 36$ Equation 2

General concept:

From equation 1 and 2, we get

Square of an odd number is odd

Square of an even number is even.

Example 2 :

Sum of two odd numbers is even

Solution:

Particular concept:

1+1=2

1+3=4

1+5=6

3+5=8

General concept:

In the above we conclude that sum of two odd numbers is even

Merits Of Inductive Method

- It enhances self confident
- It is a psychological method.
- It is a meaningful learning
- It is a scientific method
- It develops scientific attitude.
- It develops the habit of intelligent hard work.
- It helps in understanding because the student knows how a particular formula has been framed.
- Since it is a logical method so it suits teaching of mathematics.
- It is a natural method of making discoveries, majority of discoveries have been made inductively.
- It does not burden the mind. Formula becomes easy to remember.
- This method is found to be suitable in the beginning stages. All teaching in mathematics is conductive in the beginning.

Demerits Of Inductive Method

- Certain complex and complicated formula cannot be generated so this method is limited in range and not suitable for all topics.
- It is time consuming and laborious method
- It is length.
- It's application is limited to very few topics
- It is not suitable for higher class
- Inductive reasoning is not absolutely conclusive because the generalization made with the help of a few specific examples may not hold good in all cases.

DEDUCTIVE METHOD

Deductive method is based on deduction. In this approach we proceed from general to particular and from abstract and concrete. At first the rules are given

and then students are asked to apply these rules to solve more problems. This approach is mainly used in Algebra, Geometry and Trigonometry because different relations, laws and formulae are used in these sub branches of mathematics. In this approach, help is taken from assumptions, postulates and axioms of mathematics. It is used for teaching mathematics in higher classes.

Deductive approach proceeds form

- General rule to specific instances
- Unknown to know
- Abstract rule to concrete instance
- Complex to simple

Steps in deductive approach

Deductive approach of teaching follows the steps given below for effective teaching

- Clear recognition of the problem
- Search for a tentative hypothesis
- Formulating of a tentative hypothesis
- Verification

Example 1:

Find $a^2 \text{ X } a^{10} = ?$

Solution:

General : $a^m \text{ X } a^n = a^{m+n}$

Particular: $a^2 \text{ X } a^{10} = a^{2+10} = a^{12}$

Example 2:

Find $(102)^2 = ?$

Solution:

General: $(a+b)^2 = a^2+b^2+2ab$

Particular: $(100+2)^2 = 100^2 + 2^2 + (2 \text{ x } 100 \text{ x } 2)$

$= 10000+4+400= 10404$

<u>Merits Of Deductive Method</u>

- It is short and time saving method.
- It is suitable for all topics.

- This method is useful for revision and drill work
- There is use of learner's memory
- It is very simple method
- It helps all types of learners
- It provides sufficient practice in the application of various mathematical formulae and rules.
- The speed and efficiency increase by the use of this method.
- Probability in induction gets converted into certainty by this method.

Demerits Of Deductive Method

- It is not a psychological method.
- It is not easy to understand
- It taxes the pupil's mind.
- It does not impart any training is scientific method
- It is not suitable for beginners.
- It encourages cramming.
- It puts more emphasis on memory.
- Students are only passive listeners.
- It is not found quite suitable for the development of thinking, reasoning, and discovery.

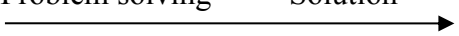
4 B- Activity centered methods

Problem solving method

Introduction

- Scientific method involves reflective thinking, logical reasoning, scientific enquiry, practical skills and scientific attitude.
- Scientific method is basically problem solving method.
- According to James, problem solving is an educational device when teacher and students cautiously, with planning, purposefully attempt to explain or solve some educationally (scientifically) significant difficulty (problem).
- According to Risk, problem solving may be defined as planed attack upon a difficulty (problem) for purpose of finding proper solution.

Problem "Problem solving" Solution



Characteristics

- Problem should be according to the capacity of students.
- Problem should be related to previous knowledge of the student.
- Problem should have practical and educational value.
- Language of problem should be simple and clear.
- Problem should neither be too difficult nor too easy.
- It should be related to the syllabus.
- Problem should be challenging to students.
- It should fit within the administrative time table of the school.

Steps of problem solving method

1) Sensing the problem
    - A situation should be provided to student wherein students can feel to inquire about something and feel to ask a question.
    - Teacher can also give problem to student and stimulate student to find out the solution.

2) Defining the problem
    - Problem should be defined in clear, simple and scientific language.
    - Teacher should help students in framing the statement of the problem.

3) Analysing the problem
    - Student will now study words and phrases in detail, understand the whole problem properly and then start solving the problem systematically.

4) Collection of data
    - After analysing problem, students should refer different books, internet etc for collecting data.
    - They should also collect data from field related to the problem.

5) Interpretation of data
    - After collecting data, data should be organised on the basis of similarities and differences.
    - Collected data can be well organised with the help of tale and graph.
    - Teachers should help students in organising and interpreting data.

6) Formulation of hypothesis
    - Students are asked to form tentative hypothesis.
    - Hypothesis is probable solution for the problem.

7) Selection and testing of hypothesis
   - Students, after experimentation reject one hypothesis and accept another.

8) Drawing conclusion
   - On the basis of results and selected hypothesis, conclusion can be made.

9) Application of generalization (conclusion) to new situation
   - After solving problem, the conclusion or generalization can be applied by the student in his or her real life or a classroom situation.

Advantages

- By problem solving method, children get trained to solve problems of real life.
- This method stimulates critical thinking, logical thinking and reasoning ability of students.
- It helps in making students self reliable and resourceful.
- Students develop confidence.
- It motivates students to gather data and information independently.
- Students gradually develop ability of analysing problem.
- They develop patience and tolerance.
- It is based on the principle of "learning by doing".
- IT gives a break from boredom due to traditional teaching- learning process.
- Students develop skill of observation.

Disadvantages

- It is lengthy and time consuming method.
- All topics cannot be covered.
- It is applicable only for higher standard classes.
- It is difficult part for the teachers to organize content based on this method.
- It is not suitable for slow learners.
- Text books do not always contain problems related to real life.
- Students have to give independent effort to find put a solution which is not possible in case of every student.
- Organizing data, finding hypothesis and interpreting data is at times difficult at school level.
- This method demands well trained and experienced teachers.
- It requires proper infrastructure, apparatus etc to carry out experimentation which can be expensive.

Conclusion

- Problem solving method is a progressive method and an innovative teaching-learning process.

- Though it has certain demerits it can be overcome by selecting appropriate content which can fir into problem solving method.

2. Lecture cum Demonstration Method

Introduction

- This method includes demonstration of certain experiment by teacher along with proper explanation of that experiment.
- This method is devised to overcome demerits of lecture method.
- This method is according to maxim of teaching from concrete to abstract.
- Students see actual apparatus and experiment. Therefore, they feel interested in learning.

Steps involved in Lecture cum Demonstration method

1) Planning and preparation
   - Teacher should plan lesson properly i.e. lesson plan.
   - Teacher should collect and arrange all apparatus required for demonstration.
   - Do the rehearsal of demonstration.

2) Introduction of lesson
   - Teacher should introduce lesson before starting demonstration.
   - Teacher may use previous knowledge of student related to experiment.

3) Presentation of subject matter
   - While demonstration of experiment the teacher should explain laws, principle or theory related to experiment.
   - Language of experiment should be simple and clear yet scientific.

4) Experimentation
   - Demonstration should be properly placed so that all students are able to observe it.
   - Time management should be kept in mind.

5) Black Board work
   - Teacher should make use of black board to summarize principles of experiment, to draw diagram, write difficult scientific words etc.

Advantages

1) This method includes theoretical and practical knowledge.
2) It gives clear, undoubted and permanent knowledge.
3) There is semi-active participation of students if teacher asks students to perform some part of demonstration.
4) Multisensory communication is used which leads to interesting successful teaching-learning process.

5) It leads to better understanding of subject as concrete things are shown to students.
6) This method is useful in explaining experiments which are costly or which includes corrosive reagents.
7) It is time saving.
8) This method is suitable for all types of students.
9) This method helps teacher to gain student's attention.
10) It develops reasoning ability and critical thinking ability of students.

Disadvantages

1) This method is majorly teacher dominating. Students are passive and observing demonstration.
2) Students do not get practical experience.
3) Students are not able to develop their practical skills as they are not involved in doing experiment.
4) This method is not psychologically sound for students as 'learning by doing' is not followed.
5) Students do not get opportunity to do planning, construction, innovation etc. as whole experiment is demonstrated by teacher.
6) On teacher's part lot of preparation is required. Teachers have to invest more time in this method.
7) If students are large in number this method is not that effective as all students will not be able to observe demonstration.
8) This method demands well- experienced and well trained teacher.
9) Students sitting in last benches can be inattentive if they are not able to observe demonstration.

Suggestions

1) Students should be made to stand in semi- circular manner so that each one gets opportunity to observe demonstration.
2) Small groups of students should be made and then experiments should be conducted.
3) Mirror should be placed behind the demonstration table so that all students are able to observe the demonstration.
4) Rooms should be well ventilated and properly lighted.
5) Apparatus should be arranged in proper manner i.e. left to right or right to left.
6) Chemicals which are to be used in demonstration should be properly checked.
7) Experiment set up should be according to level of student. It should not be too lengthy.
8) Experiments should be properly connected or related to the theory, principle or law.
9) Teacher should have good control over class.

4 C- Techniques of teaching Mathematics

In mathematics teaching, teaching techniques are such aids which are used to make the lesson interesting, to explain the content and to remember it by heart during teaching learning process. Techniques are not directly linked with the teaching objectives, but they are linked with the teaching methods.

Drill in Mathematics

Drill and exercises occupy an important place in mathematics teaching and learning. Drill work is based on the psychological principles such as learning by doing and law of exercise. Drill affords a convenient and fairly efficient medium for the rapid memorisation of details and the automatization of processes. Drill must be recognised as an essential means of attaining some of the desired controls. A strong emphasis upon concepts and meanings must be regarded as essential for understanding.

Drill provides an opportunity of self -learning and improvement. The speed and the accuracy in mathematics cannot be possible without drill work.

How to Make Drill Effective

Following points should be considered--:

1) Drill must be most effective and well-motivated.
2) Drill exercises should be conducted in such a manner that pupils can work at different rates and at a different levels according to their abilities.
3) Drill exercises should be brief and distributed over a period of time.
4) Variety of problems will make the drill interesting.
5) The drill work should be progressively more challenging.
6) Drill exercises should contain enough material to keep all the students profitably occupied throughout the drill period.
7) In order to be most effective, drill exercise must be specific.
8) Efforts should be made to detect mistakes in children's wprk and eliminate them at the outset.
9) The students should be enabled to take pleasure in drill work.
   It is of extreme importance to supervise closely the initial work of the students on any new process.
10) After the children have done the practice , the teacher should try to elicit a summary of what has been learned in the classroom.
11) The summary can also be developed by asking questions related to the concept developed in the classroom.

Advantages of Drill Work

1) Learnt material can be retained for a longer time.
2) It is a good technique of learning for beginners.

3) Speed of the learning material can be adjusted according to need.
4) Accuracy of learnt material can be improved.
5) Memory of the child can be checked.
6) Pronunciation of the child can be corrected.
7) It is very economical device of learning and teaching.
8) It is a less time -consuming technique of learning.
9) Immediate reinforcement through practice and application is desirable.

Disadvantages of Drill Work

1) It is not suitable for all topics.
2) Drill work creates disturbance in the other classes.
3) It is not effective without good and clear voice.
4) Sometimes drill becomes an exercise in academic futility and no one benefits.
5) Careful questioning by the teacher is usually needed in drill work.

Review

Review means to view again. Review aims at recalling the past experiences to produce better retention.

The mental process of going over material body is ordinarily term as review.

Sometimes review in mathematics is identified with drill work because they are both characterised by repetition and both aim at the fixation of concepts, relationships or reactions.

Review aims not only at the fixation and retention of details but also at the thoughtful organisation of the important things in a unit or a chapter in order that the relationship of the various parts to each other and to the whole unit maybe understood clearly.

The children need to be taught that they have to review material just as they need to be taught how to study. They cannot review effectively without definite instructions. The task of helping children to plan their review work is a responsibility of the teacher.

Purposes of Review

1) To fix the knowledge.
2) To have better understanding.
3) To develop new interest in old materials.
4) To prepare background for new learning.
5) To introduce new elements.

Importance of Review

1) The review strategy is helpful in recalling the facts and maintaining the sequence of material learnt.
2) It is very useful to consolidate the learnt material in the mind.

3) When data are organised in tabular forms, graphs, charts, etc. there takes place a good deal of reflective thinking.

4) During the learning process of mathematics, certain facts, formulae, principles of geometrical figures are to be retained.

5) In order to minimize forgetting of the acquired knowledge, a systematic review is often needed.

Assignment in Maths

Assignment is the work given to the students either before the lesson or after the lesson and it may be completed at school or at home.

The child undertakes upon himself the responsibility of carrying out the work assigned to him.

Purposes of Assignment

1) To solve mathematical problem and riders based on proposition.

2) To develop the skill of problem solving and the habit of practice.

3) To apply the mathematical knowledge in solving problem.

4) To correlate the experiences and previous knowledge of the pupil

5) To motivate students, clears up misunderstandings and develop insight.

6) To create interest in mathematics.
7) To carry out some mathematical projects.
8) To prepare illustrations for a topic.

Steps for Preparing Assignment

In preparing the assignment following steps should be taken:

1) **Plan the Assignment work**: The kind of activity provided to the students should be in accordance with their mental level. Psychology of students should be considered. The quantity of the work given should neither be too short or too more.

2) **Provide Proper Guidelines to the Students:** Students should be given proper guidance so that the work is completed efficiently by them. They should be provided with hints, clues, notes, references etc.

3) **Supervise the work:** The teacher should supervise the students to see that they are making use of correct material.

4) **Evaluate the Assignment work:** The teacher should carefully check the assignment and point out the mistakes made by the students.

Characteristics of Assignments

An assignment should have the following characteristics:

1) It should be clear and specific.
2) It should follow psychological principle.
3) It should have co-relation with students' experiences and previous knowledge.
4) It should be motivating.
5) It should consider individual difference.
6) It should stimulate and supplement learning activity.

Examples of Assignment Work

1) Constructing figures, bisecting a line segment, tangent to a circle.

2) Collecting historical background of any mathematical content.

3) Prepare charts,specimens on important topics.

4) Visiting Post-Office, Banks and studying their transactions.

Merits of Assignment

1) It fosters will and incentives within the students. The students get the power to face boldly the problems of his future life.

2) The pupil prepares his own material and he learns to handle and make use of different types of books and he seeks knowledge from different sources.

3) As it presupposes a great deal of written work he develops lucid and logical expression.

4) Students do not simply memorise fragments of factual knowledge, but whatever they learn is actively shared by them,

5) Students learn by their own efforts.

UNIT 5: Learning Resources

5A - Mathematic laboratory objectives and significance

Introduction of mathematic laboratory:-

Mathematics teaching in today's school is textbook dominated, concerned with the manipulation of symbols and all too often largely removed from the real world of the child endless repetition, meaningless memorization and general lack of interest are other few reason that attribute negative attitudes towards mathematics. NCERT had initiated to make teaching and learning of mathematics at school stages activity based and experimentation oriented. The laboratory approach allow pupil to set up mathematical experiment for the purpose of discovering some mathematical principles, patterns or process. These activities may be carried out by the teacher or the students to explore to learns, to stimulate interest and develop favourable altitudes towards mathematics.

Definition:-

- The maths lab provides an opportunity for the students to discover mathematics through doing.
- The mathematics laboratory is a place where anybody can experiment and explore patterns and ideas. It is a place where one can find a collection of games, puzzles, and other teaching and learning material
- Mathematics laboratory is a place to enjoy mathematics through informal exploration.
- It is a place where anyone can generate problems and struggle to get an answer.

Objectives:-

A. Knowledge and understanding

- know and demonstrate understanding of the concepts from the five branches of mathematics (number, algebra, geometry and trigonometry, statistics and probability, and discrete mathematics)
- use appropriate mathematical concepts and skills to solve problems in both familiar and unfamiliar situations including those in real-life contexts
- Select and apply general rules correctly to solve problems including those in real-life contexts.

B. Investigating patterns

- select and apply appropriate inquiry and mathematical problem-solving techniques
- recognize patterns
- describe patterns as relationships or general rules
- draw conclusions consistent with findings
- Justify or prove mathematical relationships and general rules.

C. Communication in mathematics

- use appropriate mathematical language (notation, symbols, terminology) in both oral and written explanations
- use different forms of mathematical representation (formulae, diagrams, tables, charts, graphs and models)
- Move between different forms of representation.

D. Reflection in mathematics

- explain whether their results make sense in the context of the problem
- explain the importance of their findings
- justify the degree of accuracy of their results where appropriate

Significance:-

1. Mathematics laboratory is activity centred and a child is placed in problem solving situation through self-exploration and discovery. He provides a solution based on his experience, needs and interest. Some of the ways in which a mathematics laboratory can contribute to the learning of the subject are.

2. It provides an opportunity to understand and internalize the basic mathematical concepts through concrete objects and situations.

3. It enables the students to verify or discover several geometrical properties and facts using models or by paper cutting and folding technique.

4. It helps the students to build interest and confidence in learning the subject.

5. The laboratory provides opportunity to exhibit the relatedness of mathematical concepts with everyday life.

6. It provides greater scope for individual participation in the process of learning and becoming autonomous learners.

7. It provides scope for greater involvement of both the mind and the hand which facilitate cognition.

8. The laboratory allows and encourages students to think, discuss with each other and the teacher and assimilates the concepts in a more effective manner.

9. It enables the teacher to demonstrate, explain and reinforce abstract mathematical ideas by using concrete objects, models, charts, graphs, pictures, posters etc.
10. Mathematics lab helps develop meta-cognitive abilities. As the student can rethink and rework the problem and solution several times without any interference from time constraints.

11. It builds up inherent interest and confidence in the student towards learning and doing mathematics.
12. Construction and demonstration of working model in geometry help in grasoing the geo metrical facts.

13. It widens the experiential base and lays groundwork for later learning of new areas of mathematics and helps to make appropriate connections between the known and unknown in mathematics.

14. Since theoretical concepts can be easily clarified in the laboratory so it saves time and energy of the teachers and students.

15. A students can acquire the real knowledge and heuristic altitude through practical work in the laboratory.

16 .students develops a many good habits such as co-operation, resourcefulness, initiative self-confidence, self –reliance etc.

Mathematics Club :

Introduction

Mathematics club gives opportunities to the students to showcase their mathematical skills. The main objective of the club is to explore mathematics beyond textbook and apply it in the today's situation. The club conducts competitions which give opportunities for the students to solve challenging questions, puzzles and riddles. We also encourage the students to be more creative by organizing model making contest.

In an attempt to provide opportunities for experiential learning to the students, the club organises its annual mathematics fest year after year. Teaching sessions are conducted where the students are taught the concept through PowerPoint presentations made by the members themselves.

Mathematics club:

The meeting place where a number of persons meet to discuss something or to study some problems is known as club and where students of mathematics and discuss together is called mathematics club. Mathematics club plays an important role in motivating the students to learn mathematics with interest and involvement. Clubs provide excellent opportunities for students to break away from classroom teaching of rigid and structured classroom environment which is informal in nature. students choose their own activities and pursue them in a free and relaxed manner club provides a lot of freedom of expression for the students and it supplements class room learning.

Need and importance of mathematics club :

1. It inculcates the habit of self-study and work among the students.

2. It helps the child in the proper utilization of leisure time with some purposeful activities.

3. It provides a less formal environment of study as compared to usual classroom teaching.

4. It helps in developing heuristics and problem solving attitude among the students.

5. It helps the student to imbibe social qualities like cooperation, tolerance, adjustment and open - mindedness as they work in groups.

Mathematics club bearers:

* Patron

* Sponsor

* Members

* Executive committee

a. Chairman

b. Secretary

c. Assistant secretary

d. Treasurer

e. One or two representatives from each group.

Duties of the Club- bearers :

Patron: Mathematics club have the head of the institution as its patron. He or she is responsible for providing all facilities to the club.

Sponsor: One of the senior mathematics teacher can be the sponsor of the mathematics club. He or she will take initiative to the club, motivates the students to plan the activities.

Members: All the mathematics students of the school can be members of the club. Those students who are interested in mathematics can be the members.

Executive committee:

An executive committee should be formed either through election or nomination from among the students. Members of the executive committee have to extend their full cooperation in planning and carrying out the program's of the mathematics club.

Activities of the Mathematical Club:

1. Organizing mathematical exhibition or fairs.

2. Organizing lectures on important topics of mathematics by eminent scholars and teachers.

3. Celebrating historical dates and events of mathematical significance.

4. Organizing inter-School contests and quiz competition topics of mathematical interest.

5. Publication of newsletter and bulletin of the club.

Conclusion : Extracurricular activities are vital part of the school program and social life for most students and they learn as much outside of the classroom as they do inside. In each mathematical activity student bear substantial responsibility for leadership, and many learn important leadership skills in the club

5B- Essential Characteristics of a good mathematics textbook:

Introduction:

In the teaching-learning process, the text-book occupies an important place. There is a saying "As is the text-book, so is the teaching and learning". A good text-book can even replace class-room teaching. The mathematics text-book should aim at aiding the pupils in the development of their personalities, in developing open mindedness, developing appreciation and understanding of nature and not merely stuffing their minds with facts.

Characteristics of a good mathematics text-book :

1. The author: A good text-book is judged, at face, by the author, his qualification and experience.

2. **Mechanical features of the text-book:**

- The print and paper used and the binding of the text-book should be attractive. It should be hard and durable.
- The printing should be clear, legible and appropriately spaced.
- The book should be well-illustrated with diagrams, sketches and pictures.
- The size of the print, the language and experiments discussed should suit the age of the child and standard of the child.

3. The subject matter-its nature and organisation:

- The subject-matter should be developed as far as possible in psychological sequence. Care must be taken of the mental growth and interest of pupils.
- There should be consistency of the subject-matter and the text-book should satisfy the objectives of mathematics teaching.
- Each chapter should begin with a brief introduction and end with a summary.

- Subject-matter should lead to the inculcation of scientific attitudes, disciplinary and cultural values.
- Each chapter should contain assignments at the end.
- During treatment of subject-matter, numerical examples should find place where necessary.
- Headings and sub-headings are given in bold letters.
- Each text-book should contain detailed Table of Contents and an index.
- The language of the book should be simple, clear, lucid, scientific and precise. The English equivalents of the terms should be always given in brackets.
- The text-book should give suggestions for improving scientific apparatus.
- Examples in the text-book should be given from local environment and from life experience.
- During the treatment of mathematics subject in the text-book, care should be taken to see that it is correlated with other subjects like craft, social environment and physical environment.
- Each text-book should be accompanied by a laboratory manual.

5 C - Digital resources for teaching mathematics (Geogebra And Virtual Manipulatives)

GEOGEBRA

GeoGebra is an interactive geometry, algebra, statistics and calculus application, intended for learning and teaching mathematics and science from primary school to university level.

GeoGebra is available on multiple platforms with its desktop applications for Windows, Mac OS and Linux, with its tablet apps for Android, iPad and Windows, and with its web application based on HTML5 technology.

Its creator, Markus Hohenwarter, started the project in 2001 at the University of Salzburg, continuing it at Florida Atlantic University (2006–2008), Florida State University (2008–2009), and now at the University of Linz together with the help of open-source developers and

translators all over the world. After a successful Kickstarter campaign, GeoGebra expanded their offerings to include an iPad, an Android and a Windows Store app version.

GeoGebra is interactive mathematics software for learning and teaching mathematics and science from primary school up to university level. Constructions can be made with points, vectors, segments, lines, polygons, conic sections, inequalities, implicit polynomials and functions. All of them can be changed dynamically afterwards. Elements can be entered and modified directly via mouse and touch, or through the Input Bar. GeoGebra has the ability to use variables for numbers, vectors and points, find derivatives and integrals of functions and has a full complement of commands like Root or Extremum. Teachers and students can use GeoGebra to make conjectures and to understand how to prove geometric theorems.

Its main features are:

- Interactive Mathematics Environment
- Built-in spreadsheet with extended skills
- Built-in CAS
- Allows scripting
- Large number of interactive learning and teaching resources at GeoGebraTube

GeoGebra activities - Mathematics for Teaching:

- The Pythagorean Theorem Puzzle
- Making connections: Square of a sum
- Teaching mathematics with GeoGebra
- What is a coordinates system?
- Constructing polygons with equal area
- Problem on proving perpendicular segments
- Teaching with GeoGebra – Investigating coordinates of points
- Teaching simplifying and adding radicals
- GeoGebra, Calculator, and Mathematics
- GeoGebra and Learning Mathematics
- Problem Solving Involving Quadrilaterals
- Teaching the concept of function

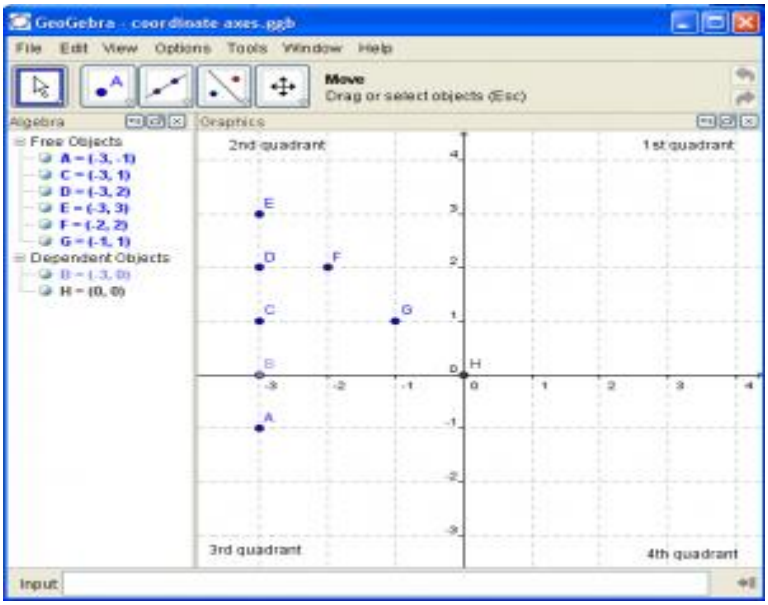
The topic includes four GeoGebra activities:

Activity 1 – What are coordinates of points?

Activity 2 – What are the coordinates of points under reflection in x and y axes?

Activity 3 – How to describe sets of points algebraically Part 1?

Activity 4 – How to describe sets of points algebraically Part 2?



On the one hand, GeoGebra is an interactive geometry system. You can do constructions with points, vectors, segments, lines, polygons and conic sections as well as functions while changing them dynamically afterwards.

On the other hand, equations and coordinates can be entered directly. Thus, GeoGebra has the ability to deal with variables for numbers, vectors and points. It finds derivatives and integrals of functions and offers commands like Root or Vertex.

1) **GeoGebra's user interface**

Using the provided geometry tools in the Toolbar you can create geometric constructions on the Graphics View with your mouse. At the same time the corresponding coordinates and equations are displayed in the Algebra View. On the other hand, you can directly enter algebraic input, commands, and functions into the Input Bar by using the keyboard. While the graphical representation of all objects is displayed in the Graphics View, their algebraic numeric representation is shown in the Algebra View. In GeoGebra, geometry and algebra work side by side.

The user interface of GeoGebra is flexible and can be adapted to the needs of students. If you want to use GeoGebra in early middle school, you might want to work with a blank sheet in the Graphics View and geometry tools. Later on, you might want to introduce the coordinate system using a grid to facilitate working with integer coordinates. In high school, you might want to use algebraic input in order to guide your students through algebra on into calculus.

Apart from the Graphics and Algebra View, GeoGebra also offers a Spreadsheet View, a Computer Algebra (CAS) View, as well as a second Graphics View. These different views can be shown or hidden using the View menu. For quick access to several predefined user interface

configuration, you may want to try the Perspectives sidebar by clicking the bar to the right of the Graphics View.

2) **Basic Use of GeoGebra**

Activate a tool by clicking on the button showing the corresponding icon. Open a toolbox by clicking on the lower part of a button and select another tool from this toolbox.

Note: You don't have to open the toolbox every time you want to select a tool. If the icon of the desired tool is already shown on the button it can be activated directly.

Toolboxes contain similar tools or tools that generate the same type of new object. Click on the icon at the right of the Toolbar to get help on the currently active tool.

3) **Saving GeoGebra Files**

Open the File menu and select Save.

Select the folder GeoGebra_Introduction in the appearing dialog window.

Type in a name for your GeoGebra file.

Click Save in order to finish this process.

4) **Creating drawings with GeoGebra**

Click on the arrow at the right side of the Graphics View and select Basic Geometry from the Perspectives Sidebar. Right-click (MacOS: Ctrl-click) on the Graphics View and choose Grid to show the grid lines

5) **Drawing Pictures with GeoGebra**

Use the mouse and the following selection of tools in order to draw figures in the Graphics View

Move: Drag a free object with the mouse.

Line: Click on the Graphics View twice or on two already existing points.

Segment: Click on the Graphics View twice or on two already existing points.

Delete: Click on an object to delete it.

Undo / Redo: Undo / redo a construction step by step (on the right side of the Toolbar).

Move Graphics View: Click and drag the Graphics View to change the visible part.

Zoom In / Zoom Out: Click on the Graphics View to zoom in / out.

Advantages

- Dynamic mathematics software (DMS) provides new opportunities for using both computer algebra system (CAS) and dynamic geometry software (DGS).
- GeoGebra is a DMS that freely available on-line, and supplemented with a variety of dynamic worksheets.

- This software also allows students to make and test assertions and prepare for more formal proof writing. Students can build a geometric construction and simultaneously observe how changes in a formula in the algebra window are affected by the manipulation of the construction and vice versa.
- Teachers can use this software to construct interactive applets on the internet to improve students' proving abilities.
- By participating in these explorative tasks, the student will engage in realizing geometric invariants and formulating conjectures activities. As a result, students take produced arguments for granted that support to construct formal proofs.
- This software can be downloaded freely at the following website: http://www.geogebra.org

In short, the development of DMS provides students with many opportunities to explore and discover mathematics concepts according to their own individual needs and pace. This dynamic environment could motivate students to explain their empirical conjectures using formal proofs and provide an opportunity to link empirical and deductive reasoning together.

A DMS can also be utilized to gain insight into a deductive argument, support experimentation, and thus lead to conviction**.**

Learning mathematics within a dynamic geometry software involves transitions in the learning process between figures and concepts, between perceptual activity and mathematical knowledge. Typically, a geometrical problem cannot be solved while remaining only at the perceptual level of figures on the screen. Conceptual control is needed and this requires explicit knowledge. The use of the dragging function validates procedures and is the crucial instrument of mediation between figure and concepts, perception and knowledge.

Disadvantages

GeoGebra is a complex mathematical application for those that study or work with **arithmetic, geometry, algebra and calculus**.

It's a rather complex application that's aimed strictly at those comfortable with complex maths but the advantage GeoGebra offers over similar apps is that, it provides multiple representations of objects that are all dynamically linked. The idea behind GeoGebra is to connect geometric, algebraic, and numeric representations in an interactive way.

This can be done with points, vectors, lines, conic sections. GeoGebra allows you to directly enter and manipulate equations and coordinates enabling you to plot functions, work with sliders to investigate parameters, find symbolic derivatives, and use powerful commands like Root or Sequence.

However, the complexity of the program is mind boggling for those new to such mathematical applications. It's very difficult to work out how to use from scratch although there are several very detailed tutorials to help you on your way.

GeoGebra isn't a mathematical program for the faint hearted but if you have to deal with arithmetic, geometry, algebra and calculus on a regular basis, it's very flexible.

Pros

1)It Provides multiple representations of dynamically linked objects

2)It Covers arithmetic, geometry, algebra and calculus

Cons

1)It is Complicated for beginners

Finally, we have a tool that is only limited by our own imaginations, so even if you aren't very imaginative, let your students be and let their learning soar with the endless possibilities of a tool like GeoGebra.

VIRTUAL MANIPULATIVE

A manipulative is an object which is designed so that a learner can perceive some mathematical concept by manipulating it, hence its name. The use of manipulative provides a way for children to learn concepts in a developmentally appropriate, hands-on and an experiencing way. Mathematical manipulative is frequently used in the first step of teaching mathematical concepts, that of concrete representation. The second and third step is representational and abstract, respectively. Mathematical manipulative can be purchased or constructed by the teacher.

Virtual manipulatives for mathematics are computer models of these objects. Multiple experiences with manipulatives provide children with the conceptual foundation to understand mathematics at a conceptual level. For the most part, classroom use of manipulative has involved concrete, or physical, manipulative. However, with the advent of the World Wide Web, there is a new category of manipulative—virtual. Virtual manipulatives are basically digital "objects" that resemble physical objects and can be manipulated, usually with a mouse, in the same ways as

their authentic counterparts. Virtual versions of concrete manipulative typically used in mathematics education, such as Base 10 Blocks, Cuisenaire Rods, and Tangrams, are available at no cost online. Many available virtual manipulatives are paired with structured activities or suggestions to aid implementation in the classroom.

Characteristics of ideal virtual manipulative:

- Level of difficulty can be adjusted for different students.
- Should provide qualitative feedback.
- Teachers need to provide feedback and support.
- Instructions for use should be clear.

Examples:

Pan balance number, Tangram, Math tool etc.

Applications/ Use of Virtual Manipulatives

- Manipulatives is a popular approach to help students understand the abstract concepts.
- Manipulatives enable students and teachers to represent concretely the abstract concepts that they are learning in mathematics class and to link these concepts to prior knowledge.
- Although they are used primarily in the elementary grades, they offer a useful means to introduce new concepts to all students.
- Concrete manipulatives help students with disabilities improve their understanding of the abstract symbolic language of mathematics.
- Concrete manipulatives can also be used to clarify misconceptions and build connections between mathematical concepts and representations, fostering more precise and richer understandings.
- Students who struggle in mathematics often have trouble connecting visual and symbolic representations; virtual manipulatives can make such connections explicit to students.

  For example, Pan Balance – Numbers is a manipulative that is based on the balance pans that are used with younger children to demonstrate the concept of equality. With this virtual manipulative, students enter a different number expression (e.g., 6+8 and 7+7) on each side of the balance. If the two expressions are equivalent, the pans are shown balanced (visual representation), and the equation is presented in an on-screen window (symbolic representation).

- Virtual objects can be altered in ways that concrete ones cannot, for example, the size, shape, and color of a block can be changed. Thus, in many instances, students can create more examples using virtual versus physical objects.
- Although relatively new, virtual manipulatives can support learning in mathematics for all students, including those with disabilities.

  - **Pan Balance – Numbers** is one of a series of virtual manipulatives available through the Illuminations website, that support students in investigating the concept of equivalence
  - **Tangrams** is a virtual manipulative available through the National Library of Virtual Manipulatives website). It is based on the ancient Chinese tangram blocks. The blocks can be dragged, rotated, and flipped in order to copy designs.
  - **MathTools** includes a catalogue of technology resources for math on the web. The catalogue includes hundreds of tools, lessons, and activities that are categorized and searchable by grade level and content. Each has been submitted by a registered user (registration is free and open to all). Each includes a description, the technology type (e.g., Java applet, Flash), and ratings, reviews, and discussions from other registered users of the site. Registered users can save any activities to their own "My Math Tools" portion of the site.

Advantages:

1) It teaches underlying values and skills like problem-solving, patience and attention span, critical thinking, creativity, concentration etc.

2) Students feel more capable and competent. Because they do things on their own, and discover things on their own, they feel less dependent to their teachers.

3) They will see real life applications of concepts. Rather than teaching them concepts, manipulatives allow them to literally grasp each situation and they will feel the relevance of the concepts.

4) Can keep the students occupied. Attracts a lot of attention, and can keep it as long as it is developmentally appropriate.

5) Easier for students to understand and reflect on the topic. Since everything happened under their control, it is easier for them to analyse what they did and it allows them to "play around the

concepts".

Disadvantages:

1) Costly. For virtual manipulatives, the students have to have individual computers.
2) Has to be carefully planned and takes a lot of time. There are a lot to consider when using manipulatives (both virtual and concrete). For example: Is the difficulty or complexity just right for the students? Will they get and understand the underlying concept? Is this too fun or too boring?
3) Rural areas lack facility of electricity.
4) Internet facility is not available in remote areas.
5) Trained teaches are required to make use of this facility.
6) Has to be carefully planned and is time consuming.

UNIT 6: Professional Development of Teacher

6 A- Competencies of Mathematics teacher

1. To possess a competence in some domain of personal professional or social life.
2. Competence is master to affair degree, module the condition and circumstances.
3. Mathematical competence is the ability to understand, judge, do and use mathematics in a variety of intra and extra mathematical context.
4. Lots of factual knowledge and technical skills in the same way vocabulary, orthography and grammar are necessary.
5. Mathematica competency is clearly recognisable.
   There are eight competencies which can be said to form two groups.
   First group is to do with the ability to ask and answer question in and with mathematics.

   A. Thinking mathematically (mastering mathematical modes of thought)

      1. Posing questions which are characteristic of mathematics and knowing the kinds of answers that mathematics may offer.
      2. Understanding and handling the scope and limitation of a given concept.
      3. Distinguishing between different kinds of mathematical statement.

   B. Posing and solving mathematical problems.

      1. Identifying, posing and specifying different kinds of mathematical problems. Pure or applied, open and dead or closed.
      2. Solving different kinds of mathematical problems weather posed by others or by oneself.

   C. Modelling mathematically (analysing and building model)

      1. Analysing foundation and prospective of existing models including the range and validity.
      2. Descending existing models that is translating and interpreting mode elements in terms of reality modelled.
      3. Performing active modelling in a given context.
         - Structuring the field
         - Working with the model including solving the problems
         - Validating the model internally and externally.
         - Analysing and criticising a mode in itself
         - Communicating about model and its results
         - Monitoring and controlling the entire modelling process

   D. Reasoning mathematically

      1. Following accessing chains of arguments put forward by others.
      2. Knowing what a mathematical proof and how it differs from other kinds of mathematical reasoning.

3. Uncovering the basic ideas in a given line of argument.
4. Devising formal and informal mathematical arguments and transforming heuristic arguments to valid proofs.

The second group of competency is to do with the ability to deal and manage mathematical language and tools.

E. Representing mathematical entities

1. Understanding and utilising different sorts of representations of mathematical object.
2. Understanding and utilising the relation between the different representation of same entity.
3. Choosing and switching between representations

F. Handling mathematical symbols and formalism

1. Decoding and interpreting symbolic and formal mathematical language and understanding its relation to natural language.
2. Understanding the nature and rules of formal mathematical system.
3. Translating from natural language to formal, symbolic language.
4. Handling and manipulating statements and expressions containing symbols and formulae.

G. Communicating in, with and about mathematics

1. Understanding others return, visual or oral texts.
2. Expressing oneself at different levels of theoretical and technical precision, in oral, visual or return form.

6 B- Need and Avenues of Professional Growth

Meaning of Professional Growth

When a teacher learns new techniques and new knowledge then he/she grows in his/her experience of teaching and subject knowledge. This is known as *professional knowledge.*

A number of programs have been started by different agencies to help professional growth and development of Mathematics teachers. These keep the teachers updated about new developments, researches, concepts and methods of teaching Mathematics.

Number of opportunities are provided by the Government in different programs.

AVENUES OF PROFESSIONAL GROWTH FOR MATHEMATICS TEACHERS

1.Seminars

2.Conference

3.Workshop

4.Refresher Course

5.Study Groups

6.Professional Writing

7.Summer/Winter Institution

1. SEMINAR

A seminar is a meeting where a group of people discuss a problem or topic. A seminar is a class at a college or a university in which the teacher and a small group of students discuss a topic at student level.

Example: A teacher will participate in a seminar like

A) New teaching strategies in Mathematics

b) Issues in Mathematics education

c) Career prospects

2.CONFERENCE

A formal meeting of people with a shared interest, typically one that takes place over several days.

Example: All Mathematics teachers meet in a conference and intensively discuss recent teaching-learning processes of Mathematics or teaching Mathematics for differently abled, enhancing Mathematics school laboratory.

3.WORKSHOP

A meeting at which a group of people engage in intensive discussion and activity on a particular subject or project.

Example: A workshop for Mathematics teachers on Mathematics teaching aid, improvised Mathematics apparatus, software of virtual laboratory and simulation.

4. REFRESHER COURSE

A refresher course is a training course in which people improve their knowledge or skills and learn about new developments that are related to the job they do.

Example: A refresher course on environmental education for Mathematics teachers.

Conduction of Chemistry practicals at higher secondary levels.

5. STUDY GROUPS

Various study groups at district and state level. Teachers can share their experiences on different activities.

Teaching-Learning process and difficulties faced during teaching.

Acquire professional efficiency (experience.)

Follow problem-solving approach.

Participate in refresher courses, acquire latest development in Mathematics field.

Teachers are encouraged to study various publications by NCERT.

6. PROFESSIONAL WRITINGS

This may include writings regarding new methods, techniques of teaching Mathematics or problems faced while teaching.

Instructions regarding conduction of experiments in the laboratory.

7. SUMMER/WINTER INSTITUTE

Refresh and update the knowledge of Mathematics teachers.

A) Unitary Institute

Held once a year to update the teachers in the areas of Mathematics and new technologies.

B) Sequential Institute

Training given to the best 5 teachers about the concepts of methods of Mathematics. They are prepared as resource portions.

C) Specialists

Organize all India level improvement/implementation of textual material for training college.

D) Project Technology Institute

Organized for secondary school teachers on regional basis, provides training in laboratory work, practicals and workshops.

NEED FOR PROFESSIONAL GROWTH

- Teachers should have updated knowledge to answer students' queries
- Should be acquainted with the latest strategy of evaluation
- Learn new technologies and teaching methods
- Develop scientific attitude, temper and inculcate these values in the students
- Need to acquire knowledge, ability, skills to organize Mathematics clubs, Mathematics excursions,etc
- Teachers need competency in motivating students, learning Mathematics snd applying knowledge of Mathematics in daily life
- Acquire skills of guiding students for projects and vocational guidance
- Teachers should participate and contribute in revision of Mathematics curriculum, textbooks, teaching aids, etc.

6 C- Contributions of mathematician

Pythagoras- Contributions to Mathematics

Pythagoras is often referred to as the first pure mathematician. He was born on the island of Samos, Greece in 569 BC. Various writings place his death between 500 BC and 475 BC in Metapontum, Lucania, Italy. His father, Mnesarchus, was a gem merchant. His mother's name was Pythais. Pythagoras had two or three brothers.

Pythagoras was well educated, and he played the lyre throughout his lifetime, knew poetry and recited Homer. He was interested in mathematics, philosophy, astronomy and music, and was greatly influenced by philosophy, mathematics, astronomy, philosophy and geometry.

Pythagoras settled in Crotona, a Greek colony in southern Italy, about 518 BC, and founded a philosophical and religious school where his many followers lived and worked. The Pythagoreans lived by rules of behaviour, including when they spoke, what they wore and what they ate. Pythagoras was the Master of the society, and the followers, both men and women, who also lived there, were known as mathematikoi. They had no personal possessions and were vegetarians.

Pythagoras believed:

- All things are numbers. Mathematics is the basis for everything, and geometry is the highest form of mathematical studies. The physical world can understood through mathematics.
- Numbers have personalities, characteristics, strengths and weaknesses.
- Certain symbols have a mystical significance.

Contributions of Pythagoras:

1. The sum of the angles of a triangle is equal to two right angles.
2. The theorem of Pythagoras - for a right-angled triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides.
3. Constructing figures of a given area and geometrical algebra. For example solving various equations by geometrical means.
4. The discovery of irrational numbers is attributed to the Pythagoreans.
5. The five regular solids (tetrahedron, cube, octahedron, icosahedron, dodecahedron). It is believed that Pythagoras knew how to construct the first three but not last two.
6. Pythagoras taught that Earth was a sphere in the center of the Kosmos (Universe), that the planets, stars, and the universe were spherical because the sphere was the most perfect solid figure. He also taught that the paths of the planets were circular. Pythagoras recognized that the morning star was the same as the evening star, Venus.
7. Pythagoras studied odd and even numbers, triangular numbers, and perfect numbers. Pythagoreans contributed to our understanding of angles, triangles, areas, proportion, polygons, and polyhedra.
8. Pythagoras also related music to mathematics. He had long played the seven string lyre, and learned how harmonious the vibrating strings sounded when the lengths of the strings were proportional to whole numbers, such as 2:1, 3:2, 4:3. Pythagoreans also realized that this knowledge could be applied to other musical instruments.

EUCLID

Euclid was a great Greek mathematician born in 330 BC. Although little is known about his early and personal life, he went on to contribute greatly in the field of mathematics and came to known as the 'Father of Geometry'.

He wrote 'Elements', the most influential mathematical works of all time, which aroused interest of the Western World and mathematicians around the globe for over 2000 years. His works were heavily influenced by Pythagoras, Aristotle, Eudoxus, and Thales to name a few.

There is not much information and records relating to Euclid's personal life but historians believe that he breathed his last around 260 B.C

Career

Euclid's 'Elements' is considered as one of the most influential works in the history of mathematics.

- In his Elements, he deduced the principles of 'Euclidean geometry' from a small set of axioms. Euclid also wrote works on perspective, conic sections, spherical geometry, number theory and rigor.
- 'Elements' is a mathematical and geometric treatise consisting of 13 books. It is a collection of definitions, postulates, theorems and constructions and also the mathematical proofs of the propositions. All the 13 books cover Euclidean geometry and the ancient Greek elementary number theory.
- It also includes geometric algebra, which helps in solving many algebraic problems, including the problem of finding the square root of a number.
- It is the second oldest existing Greek mathematical treatise after Autolycus' 'On the Moving Sphere'.

Other Works

In addition to his most famous work 'Elements', there are at least five works of Euclid that have survived to this day. They are 'Data', 'On Divisions of Figures', 'Catoptrics', 'Phaenomena' and 'Optics'.

- Data: This book holds 94 propositions and basically deals with the nature and implications of "given" information in geometrical problems.
- On Divisions of Figures: Another important work of Euclid but survives only partially in Arabic translation.
- Apart from these, there are some other works attributed to Euclid, but have been lost. These are 'Conics', 'Porisms', 'Pseudaria' and 'Surface Loci'

Conclusion:

The legacy that Euclid left behind is immense. He inspired personalities such as Abraham Lincoln, who used to religiously carry "The Elements" with him everywhere and used to quote the genius in his speeches.

Euclid influenced great philosophers and mathematicians such as Newton and Descartes who propounded their philosophical works using Elucid's format and structure.

ARYABHATTA

Name He was called Arya and his surname was Bhat or Bhatta!

Place of birth He born in a place called Taregana, in Bihar.

Education qualification:-

- Ganitapada, mensuration arithmetic and geometric progressions, simple, quadratic, simultaneous, and indeterminate equations.
- Geometric/trigonometric aspects of the celestial sphere.

Contribution to mathematics:-

- Contribution in the approximation of pi - Aryabhatta is among the mathematicians who brought new deductions and theories in mathematics.
- It is speculated that Aryabhata used the word sanna (approaching), to mean that not only is this an approximation, but that the value is incommensurable or irrational.
- In Ganitapada, he gives the area of a triangle as: "for a triangle, the result of a perpendicular with the half-side is the area". He discussed 'sine' by

equation $Tx^2+Ax=AB$

$$x = \frac{\sqrt{BAT + \left(\frac{A}{2}\right)^2} - \frac{A}{2}}{T}$$

The name of ardha-jya or half-chord.

- Aryabhatiya covers arithmetic, algebra, and trigonometry.
- Like other ancient Indian mathematicians, he too was interested in finding integer solutions to Diophantine equations with the form ax + by = c; he called it the kuaka (meaning breaking into pieces) method.
- His contribution to the study of Algebra is immense. In Aryabhatiya, Aryabhata provided elegant results for the summation of series of squares and cubes through well tried formulae.
- It is probable that he might have come to the conclusion that 'pi' is irrational.
- He discussed the concept of 'sine' in his work by the name of "ardha-jya", which literally means "half-chord".

- He was the one who deduced the approximate value of pi, which he found it to be 3.14. He also derived the correct formulas for calculating the areas of triangles and circles. He also played a very important role in the formation of the table of Sine's.
- His role in the place value system - He also played a very major role in determining the first to use place value system and discovering the zero.
- He also calculated the sidereal rotation .His (this) theories and deductions formed the base of the trigonometry and algebra.

Srinivasa Ramanujan

He was born on December 22, 1887, in village in Madras state, Erode, in Tanjore district in a poor Hindu Brahmin family. He was so bright in mathematics that he was declared "child mathematician" at the age of 12 by his teachers.

1). His greatest contribution to mathematics was in the field of theory of number.

2). He was one of the greatest Masters in the field of hypergeometric series and continued fractions.

3). He wrote 120 theorems on "divergent series" in 1993 which was sent to Professor Hardy.

4). He worked on "definite integrals" and "elliptic functions".

5). He pointed out that every even integer greater than 2 is the sum of two primes.
For example : 4 = 2 + 2, 6=3+3, 8=5+3, 16=13+3, 20=17+3.....

6). He pointed out that the number of
Primes in case of large numbers may be more than 2.
For example.44 = 31 + 13 = 37 + 7 = 41 + 3 .

7). He pointed out that a prime number of the form 4n+ 1 is the sum of two squares.
For example.
4(1)+1=5=2square + 1 square,
4(3)+1=13=3 square + 2 square.....

8). his work threw light on divergent series, hyper geometric series, continued fractions, the theories of numbers, definite integrals, partition functions, ecliptic functions, fractional differentiation and highly composite numbers.

9). He remembered the distinctive characteristics of the first 10,000 integers to an extent that each one of them became his personal friend

10). He is also interested with the partition of whole numbers.
Take the case of number 3. There are three alternative ways to write it 3 + 0, 1+ 2, 1+1+1.

11). He tried to generalize this notion and developed formula for the partition of any number which can be made to yield the required result by a series of successive approximations.

12). he developed ways of calculating (pi) that where so efficient that they have been incorporated into computer algorithms, permitting expressions of (Pi) in million of digits.

13). He discovered a number of remarkable identities that imply diversibility properties of the partition function.

14). He pointed out that 1729 is a very interesting number, it is the smallest number expressible as a sum of two cubes in two different ways:
1729 = 1 Cube + 12 cube = 9 cube +10 cube

15). Dr Hardy, a great mathematician of England of his time, has marked about Ramanujan's work on number - "the elementary analysis of highly composite number is most remarkable and shows very clearly Ramanujan's extra ordinary mastery over the algebra of inequalities". Ramanujan's knowledge of mathematics was starling. Although almost completely ignorant of what had been developed, his mastery continued fraction was unequaled by any living mathematician. His work has left a memorable imprint on mathematical thoughts. In memory of this great scholar we celebrate his day of birth, 22nd December as mathematics day.